Quantum compilation and its noise resilience

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Slides for [Quantum **3**, 140 (2019)] and [NJP **22**, 043006 (2020)]

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Near-term quantum computing

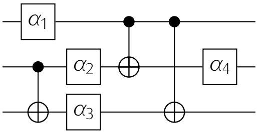
Ultimate goal: *fault-tolerant quantum computers*. But this requires many good-quality qubits: $\approx$ 1000s of physical qubits per logical qubit.

We currently have **noisy intermediate-scale quantum (NISQ)** computers:

- Limited number of qubits ($\approx$ 50–100), not fault tolerant.
- Limited connectivity between qubits.
- Limited circuit depth due to noise.

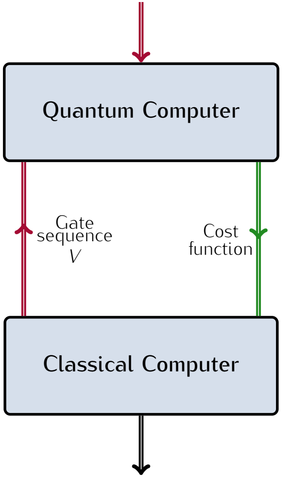
What can we do with such quantum computers?

Variational quantum algorithms [NJP 18, 023023 (2016)]



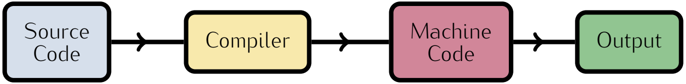
- Variational quantum eigensolver (VQE).
  [Nat. Commun. **5**, 4213 (2014)]
- Variational quantum autoencoders.
  [Quant. Sci. Technol. **2**, 045001 (2017)]
- Variational quantum state diagonalization.
  [npj Quant. Inf. **5**, 57 (2019)]

⋆ **Our variational algorithm:** *Quantum-assisted quantum compiling*
[Quantum **3**, 140 (2019)].



Compiling quantum algorithms

To run an algorithm on a quantum computer, we must *compile it*.



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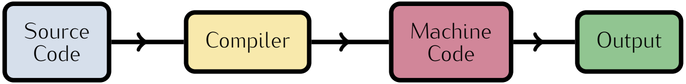
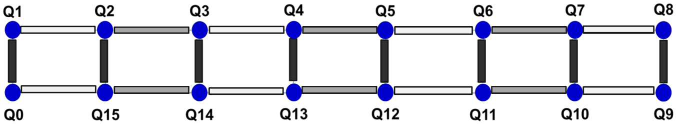
Need to adapt algorithms to NISQ device constraints:

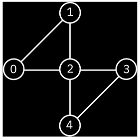
- Native gate alphabet;
- Connectivity;
- Short depth (use fewest possible gates).

Note: short depth is also important for future fault-tolerant quantum computers.

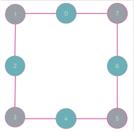




IBMQX5



IBMQX4



Rigetti 8Q-Agave

Compiling quantum algorithms

Typical compilation procedure

[IEEE Trans. CAD **27**(3), 436 (2008)]

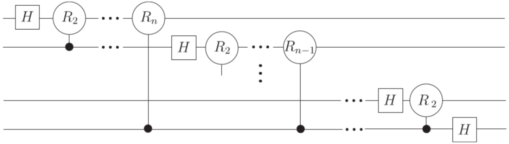
[Quantum Sci. Technol. **3**, 025004 (2018)]

Given: Unitary U as a high level gate sequence.

Step 1: Individually decompose every gate into the native gate alphabet.

Step 2: Sweep through the circuit, removing and/or combining redundant gates via, e.g., matrix identities.

Output: Gate sequence V (compilation of U).



n-qubit quantum Fourier transform

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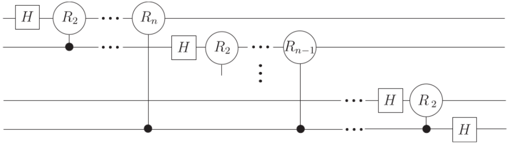

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Drawbacks

- ⋆ Step 2 can be inefficient, esp. for large circuits.
- ⋆ Requires classical simulation of the circuit in general.

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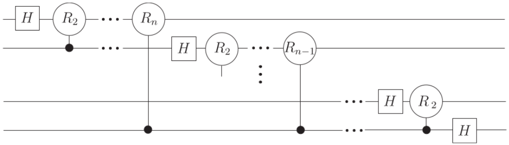

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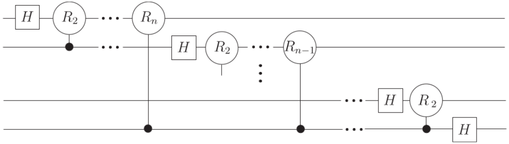

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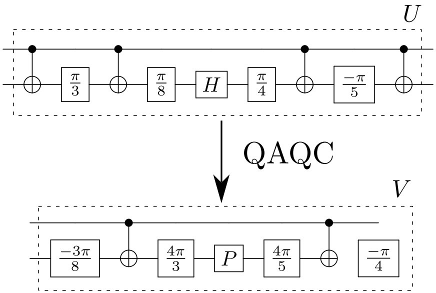
Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

QAQC performs optimal (approximate) algorithm depth compression

- U is the result of Step 1: an initial "pre-compiled", non-optimal gate sequence.
- V is the result of QAQC.

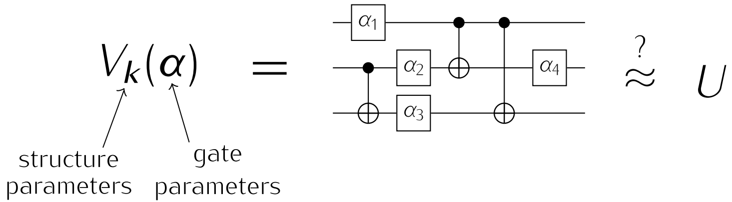


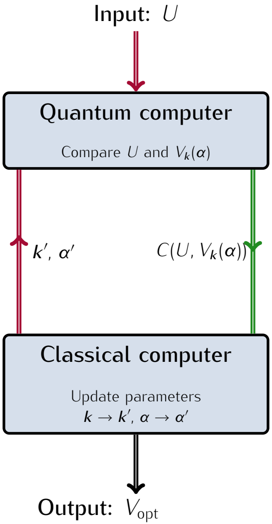
Algorithm depth compression

Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

Given: A unitary U.

Goal: find the optimal parameterized gate sequence $V_k(\alpha)$.

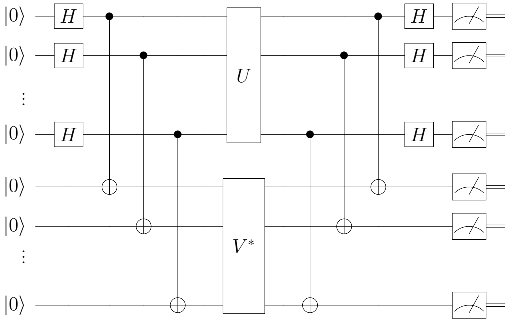




- $C(U, V_k(\alpha))$ quantifies the distance between U and $V_k(\alpha)$.
- $\alpha \to \alpha'$ via gradient descent optimizer (or non-gradient optimizer);
  $k \to k'$ via simulated annealing (or genetic algorithms).

Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

Cost function calculation on quantum computer



Hilbert-Schmidt Test (HST)

⋆ Can also compute gradient (see paper for details).

- Probability of the all-zeros outcome is

$$\frac{1}{2^{2n}}|\mathrm{Tr}[V^\dagger U]|^2.$$

The quantity is invariant under global phase.

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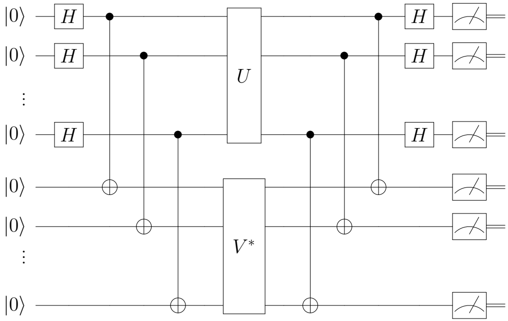

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$$C_{\mathrm{HST}}(U, V) := 1 - \frac{1}{2^{2n}}|\mathrm{Tr}[V^\dagger U]|^2.$$

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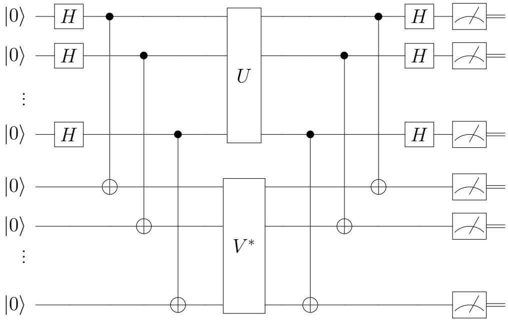

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- Related to trace distance:

$$\frac{1}{2}\left\| |\Phi_U\rangle\langle\Phi_U| - |\Phi_V\rangle\langle\Phi_V| \right\|_1 = \sqrt{C_{\mathrm{HST}}(U,V)},$$

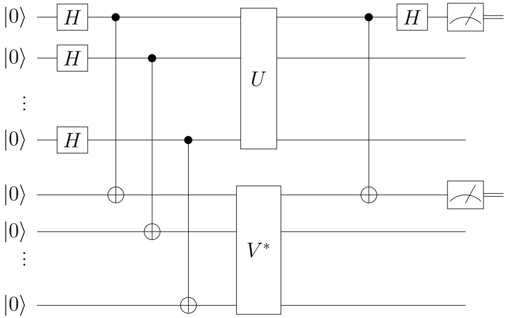
$$|\Phi_U\rangle = (\mathbb{1}\otimes U)|\Phi\rangle,$$

$$|\Phi_V\rangle = (\mathbb{1}\otimes V)|\Phi\rangle,$$

$|\Phi\rangle$: n-qubit maximally entangled state.

Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

Alternate (local) cost function for large problem sizes



Local Hilbert-Schmidt Test (LHST)

- Cost function:

$$C_{\mathrm{LHST}}(U, V) := \frac{1}{n}\sum_{j=1}^{n}(1 - \Pr[(0,0)_j]).$$

- Faithful cost function (like HST).
- Works better for optimization on large problem sizes.

⋆ Can also compute gradient (see paper for details).

Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

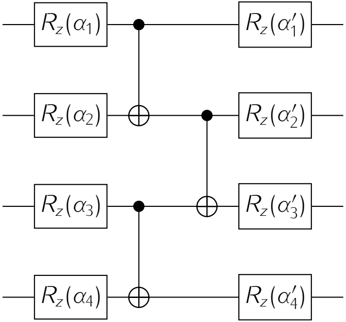
Advantages of QAQC

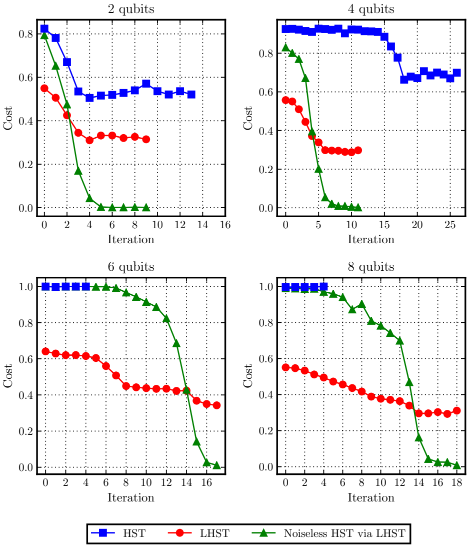
- No need to classically simulate the circuit.
- Both cost function calculations difficult to simulate classically.
  - ⋆ Both are DQC1-hard [PRL **81**, 5672 (1998)].
  - ⋆ Classical simulation of DQC1 is impossible unless polynomial hierarchy collapses to the second level [PRL **120**, 200502 (2018)].
- Holistic approach: optimizes gate parameters and also structure, and does not compile in gate-by-gate manner.
- **Not just a NISQ application:** depth compression is useful even for fault-tolerant quantum computers!

⋆ **How well does it perform, especially in the presence of noise?**

Quantum-assisted quantum compiling (QAQC) [Quantum 3, 140 (2019)]

Results on a noisy simulator



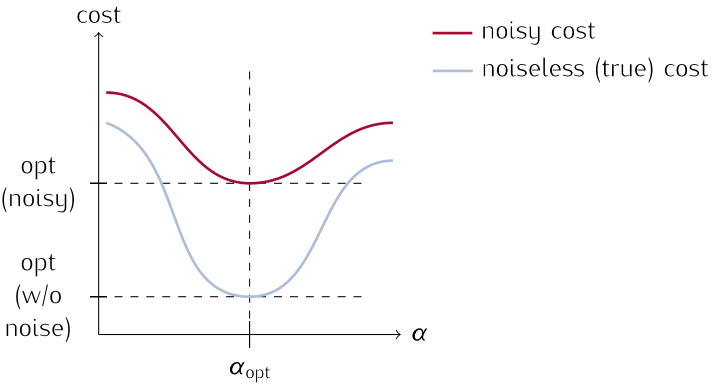


Observations

- ⋆ HST does not optimize well, but LHST does.
- ⋆ LHST optimization finds the correct minimum even in the presence of noise!

Noise resilience [NJP 22, 043006 (2020)]

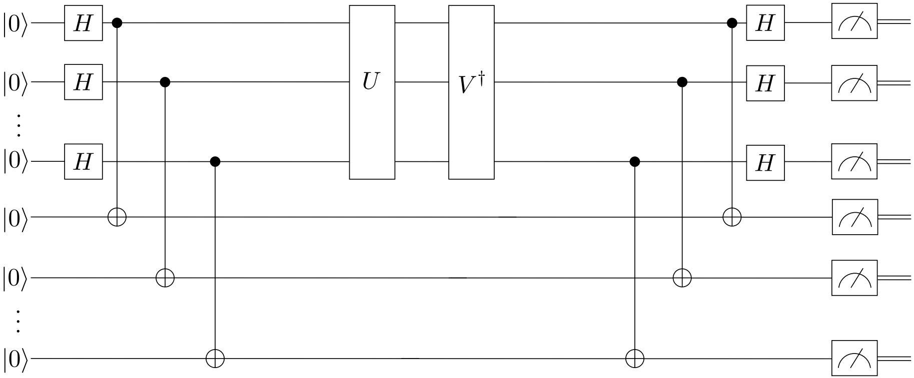
⋆ Resilience is with respect to the gate parameters: optimal gate parameters obtained via noisy optimization are also optimal without noise. So we obtain the true optimum even with noise.



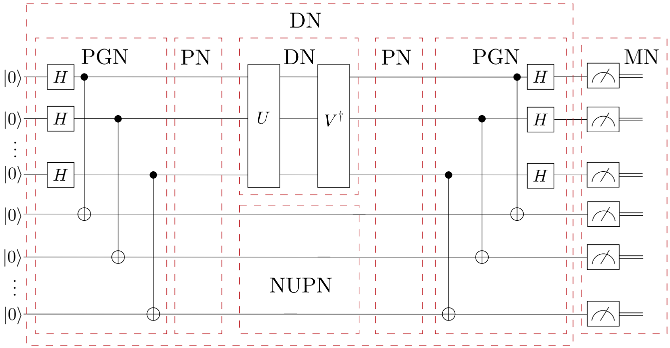
⋆ We provide proofs of noise resilience under certain noise models.

Noise resilience [NJP 22, 043006 (2020)]

Alternate HST circuit



Noise model 1

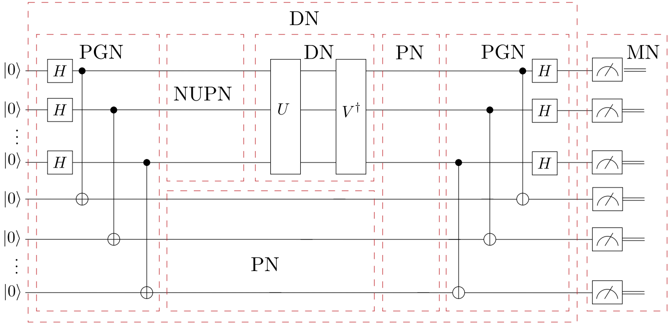


- DN: depolarizing noise
- PGN: Pauli gate noise
- PN: Pauli noise
- NUPN: non-unital Pauli noise (e.g., amplitude damping)
- MN: measurement noise

⋆ We prove noise resilience under this noise model. We can also add noise in between U and $V^\dagger$ (see paper for details.

⋆ Analogous results for LHST circuit.

Noise model 2



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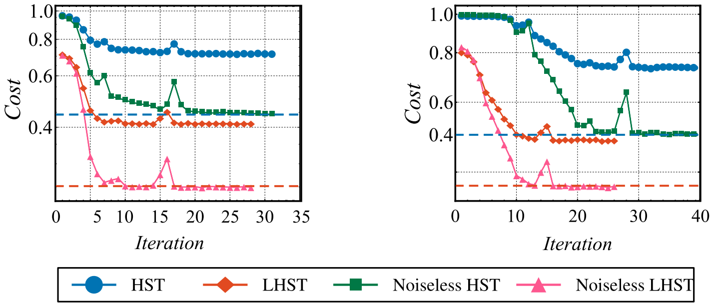
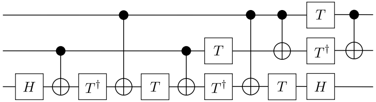
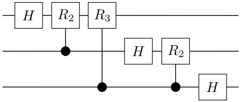
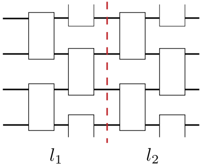
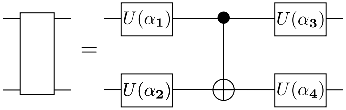
Examples











Summary & outlook

- Variational algorithm (QAQC) to compile quantum algorithms $\Rightarrow$ depth reduction.
  [Quantum **3**, 140 (2019)].
  - Circuits (HST and LHST) that compare two quantum circuits.
- Both the HST and LHST cost functions exhibit *optimal parameter resilience* under certain noise models.
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Future directions

- Investigate noise resilience of other variational algorithms.
- Algorithms for error mitigation on NISQ devices.