Policies for elementary links in a quantum network

[Quantum 5, 537 (2021)]

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Summary

Goal: Assess the performance of *near-term* quantum networks for long-distance entanglement distribution.

This work: Start at the *elementary link* level.

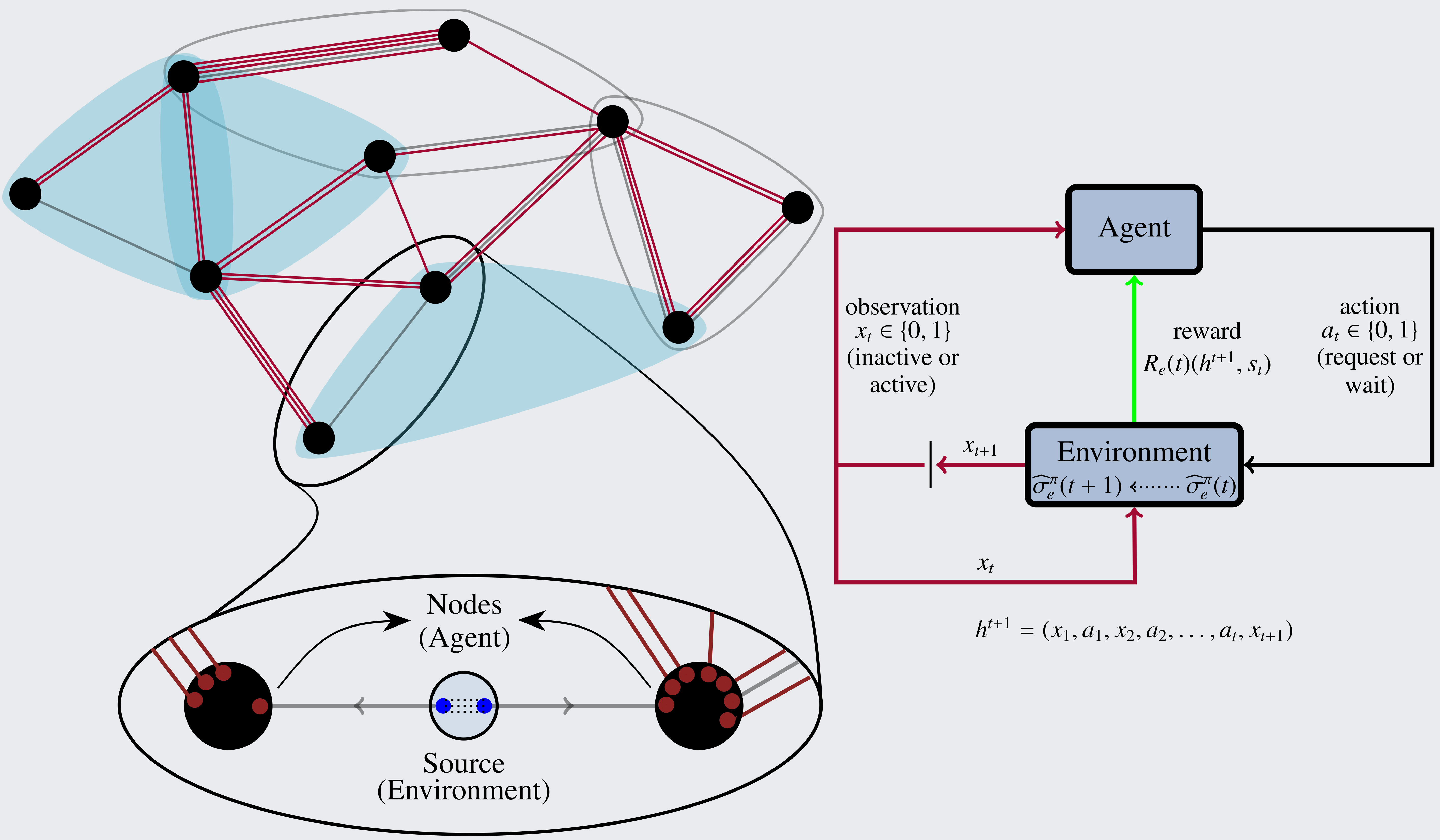
Main idea: Model elementary link protocols using *quantum decision processes*.

Results

- The expected quantum state, fidelity, and success probability of an elementary link that hold for arbitrary noise models for the quantum memories and arbitrary detection schemes.
- An algorithm to determine an optimal policy that balances fidelity and success probability.
- Complete analysis, with new results, for the *memory-cutoff policy*, a widely-used protocol for elementary links.

Outlook

- This work is the starting point for a systematic mathematical framework for analyzing quantum networks.
- Decision process can be extended to include entanglement swapping and distillation (next step for future work).



Setup

Let e be an arbitrary elementary link in the network.

System parameters

- Elementary link generation success probability p_e (depends on quantum channel from source to nodes and local measurement scheme).
- Quantum channel $\mathcal{N}_e$ describing noise of the local quantum memories.

Performance parameters (figures of merit)

- Probability that the elementary link is active at time $t \geq 1$: $\mathbb{E}[X_e(t)]_\pi$.
- Expected fidelity of the quantum state at time $t \geq 1$: $\mathbb{E}[\widetilde{F}_e(t)]_\pi$.

Results

For every elementary link e, policy π, and time $t \geq 1$:

$$\mathbb{E}[X_e(t)]_\pi = \mathrm{Tr}\left[|1\rangle\langle 1|_{X_t} \widehat{\sigma}_e^{\pi}(t)\right],$$

$$\mathbb{E}[\widetilde{F}_e(t)]_\pi = \mathrm{Tr}\left[(|1\rangle\langle 1|_{X_t} \otimes \psi)\, \widehat{\sigma}_e^{\pi}(t)\right].$$

Optimal policy: For all $T \geq 1$:

$$\max_\pi \mathbb{E}[\widetilde{F}_e(T+1)]_\pi = \sum_{x_1=0}^{1} \max_{a_1 \in \{0,1\}} w_2(x_1, a_1),$$

where

$$w_t(h^{t-1}, a_{t-1}) = \sum_{x_t=0}^{1} \max_{a_t \in \{0,1\}} w_{t+1}(h^{t-1}, a_{t-1}, x_t, a_t) \quad \forall\, 2 \leq t \leq T,$$

The optimal policy is deterministic and given by

$$d_t^{\mathrm{BR}}(h^t) := \arg\max_{a_t \in \{0,1\}} w_{t+1}(h^t, a_t) \quad \forall\, 1 \leq t \leq T.$$

Memory-cutoff policy: For all cutoffs $t^\star \in \{0, 1, 2, \dots\}$,

$$\lim_{t\to\infty} \mathbb{E}[X_e(t)]_{t^\star} = \frac{(t^\star + 1)p_e}{1 + t^\star p_e},$$

$$\lim_{t\to\infty} \mathbb{E}[\widetilde{F}_e(t)]_{t^\star} = \frac{p_e}{1 + t^\star p_e} \sum_{m=0}^{t^\star} f_e(m),$$

where f_e is the fidelity decay function (see paper for details).